CORRIGENDUM TO "A PRIORI BOUNDS FOR WEAK SOLUTIONS TO ELLIPTIC EQUATIONS WITH NONSTANDARD GROWTH" [DISCRETE CONTIN. DYN. SYST. SER. S 5 (2012), 865–878.]

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In this corrigendum we correct a lemma concerning the geometric convergence of sequences of numbers which was used in [2] as Lemma 2.1. As a consequence the statement in the main result changes a bit and the corresponding proof needs some minor different arguments to be fitted.

(a) First, we replace Theorem 1.1 in [2] by the following one:

Theorem 1.1. *Let the assumptions in (H) be satisfied. Then there exist positive constants $\alpha = \alpha(p, q_0, q_1)$ and $C = C(p, q_0, q_1, a_3, a_4, a_5, b_0, b_1, b_2, c_0, c_1, N, \Omega)$ such that the following assertions hold.*

(i) If $u \in W^{1,p(\cdot)}(\Omega)$ is a weak subsolution of (1.1), then both $\operatorname{ess\,sup}_\Omega u$ and $\operatorname{ess\,sup}_\Gamma u$ are bounded from above by

$$C\left[1+\int_\Omega u_+^{q_0(x)}dx+\int_\Gamma u_+^{q_1(x)}d\sigma\right]^\alpha.$$

(ii) If $u \in W^{1,p(\cdot)}(\Omega)$ is a weak supersolution of (1.1), then both $\operatorname{ess\,inf}_\Omega u$ and $\operatorname{ess\,inf}_\Gamma u$ are bounded from below by

$$-C\left[1+\int_\Omega (-u)_+^{q_0(x)}dx+\int_\Gamma (-u)_+^{q_1(x)}d\sigma\right]^\alpha.$$

(b) Next, we replace Corollary 1.2 in [2] by the following one:

Corollary 1.2. *Let the assumptions (H) be satisfied and let $u \in W^{1,p(\cdot)}(\Omega)$ be a weak solution of (1.1). Then $u \in L^\infty(\Omega), L^\infty(\Gamma)$ and the estimates in (i) and (ii) from Theorem 1.1 are valid.*

(c) Replace reference [32] on page 868, line 5 from bottom by the new reference [1].

(d) Now, we replace Lemma 2.1 in [2] by the following one:

Lemma 2.1. *Let $\{Y_n\}, n = 0, 1, 2, \dots,$ be a sequence of positive numbers, satisfying the recursion inequality*

$$Y_{n+1} \le Kb^n\left(Y_n^{1+\delta_1}+Y_n^{1+\delta_2}\right), \quad n = 0, 1, 2, \dots,$$

for some $b>1$, $K>0$ *and* $\delta_2 \geq \delta_1 > 0$. *If*

$$Y_0 \leq \min\left(1, (2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}\right)$$

or

$$Y_0 \leq \min\left((2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}, (2K)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_1\delta_2}-\frac{\delta_2-\delta_1}{\delta_2^2}}\right),$$

then $Y_n \leq 1$ *for some* $n \in \mathbb{N}\cup\{0\}$. *Moreover,*

$$Y_n \leq \min\left(1, (2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}} b^{-\frac{n}{\delta_1}}\right), \qquad \text{for all } n \geq n_0,$$

where n_0 *is the smallest* $n \in \mathbb{N}\cup\{0\}$ *satisfying* $Y_n \leq 1$. *In particular,* $Y_n \to 0$ *as* $n \to \infty$.

We note that Lemma 2.1 stated in [2] would have been correct if $K>1$ instead of $K>0$. However, we need in our treatment such a result for arbitrary positive K.

Now, at two places in the proof of Theorem 1.1, we need some minor changes.

(e) At the beginning of page 872 in [2] we add the following paragraph:

"Here, $(p_i^-)^*$ and $(p_i^-)_*$ are defined by, for all $i=1,\dots,m$,

$$(p_i^-)^* = \begin{cases} \frac{N(p_i^-)}{N-(p_i^-)} & \text{if } (p_i^-) < N, \\ q_0^+ + 1 & \text{if } (p_i^-) \geq N, \end{cases} \qquad (p_i^-)_* = \begin{cases} \frac{(N-1)(p_i^-)}{N-(p_i^-)} & \text{if } (p_i^-) < N, \\ q_1^+ + 1 & \text{if } (p_i^-) \geq N, \end{cases}$$

where $q_0^+ = \max_{x\in\overline{\Omega}} q_0(x)$ and $q_1^+ = \max_{x\in\Gamma} q_1(x)$ (see Section 2)."

(f) Replace the paragraph on page 876 from formula (3.23) until line 4 from bottom by the following paragraph:

"

$$\begin{aligned} Y_0 &= \int_\Omega (u-k)_+^{q_0(x)} dx + \int_\Gamma (u-k)_+^{q_1(x)} d\sigma \\ &\leq \min\left[\left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}, \left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_1\delta_2}-\frac{\delta_2-\delta_1}{\delta_2^2}}\right]. \end{aligned} \tag{3.23}$$

Relation (3.23) is clearly satisfied if

$$\begin{aligned} &\int_\Omega u_+^{q_0(x)} dx + \int_\Gamma u_+^{q_1(x)} d\sigma \\ &\leq \min\left[\left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}, \left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_1\delta_2}-\frac{\delta_2-\delta_1}{\delta_2^2}}\right]. \end{aligned} \tag{3.24}$$

Hence, if we choose k such that

$$\begin{aligned} k = &\left(1 + (16K)^{\frac{1}{q_0^-(1-\hat{\eta})}} b^{\frac{1}{\delta_1 q_0^-(1-\hat{\eta})}+\frac{\delta_2-\delta_1}{\delta_2 q_0^-(1-\hat{\eta})}}\right) \\ &\times \left(1 + \int_\Omega u_+^{q_0(x)} dx + \int_\Gamma u_+^{q_1(x)} d\sigma\right)^{\frac{\delta_2}{q_0^-(1-\hat{\eta})}}, \end{aligned} \tag{3.25}$$

then (3.24) and in particular (3.23) are satisfied. Since $k_n \to 2k$ as $n \to \infty$ we obtain

$$\operatorname*{ess\,sup}_{\Omega} u \leq 2k \quad \text{and} \quad \operatorname*{ess\,sup}_{\Gamma} u \leq 2k$$

with k given in (3.25). "

REFERENCES

[1] K. Ho and I. Sim, *Corrigendum to "Existence and some properties of solutions for degenerate elliptic equations with exponent variable"[Nonlinear Anal. 98 (2014), 146–164]*, Nonlinear Anal., **128** (2015), 423–426.

[2] P. Winkert and R. Zacher, *A priori bounds for weak solutions to elliptic equations with nonstandard growth*, Discrete Contin. Dyn. Syst. Ser. S 5, **4** (2012), 865–878.

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